system, the present problem represented by Eq. (5) cannot be reduced by integration to a second-order system. Thus this method becomes almost unique for the study of this nonlinear problem in rigid body dynamics.

References

[1] Dasarathy, B. V. and Srinivasan, P., "Class of Nonlinear Third-Order Systems Reducible to Equivalent Linear Systems," *AIAA Journal*, Vol. 6, No. 7, July 1968, pp. 1400–1401.

[2] Arnold, R. N. and Maunder, L., *Gyrodynamics and Its Engineering Applications*, Academic Press, New York, 1961, Chap. 6, pp. 104–128.

[3] Sugai, I., "Comments on Class of Nonlinear Third-Order Systems Reducible to Equivalent Linear Systems," *AIAA Journal*, Vol. 7, No. 1, Jan. 1969, p. 190.

[4] Dasarathy, B. V. and Srinivasan, P., "A New Approach to the Study of Nonlinear, Nonautonomous Systems," *Journal of Sound and Vibration*, Vol. 8, No. 1, July 1968, pp. 103–107.

[5] Jorgenson, P. S., "Theory of Gyroscope," *Gyroscopes: Theory and Design*, edited by P. H. Savet, McGraw-Hill, New York, 1961, Chap. 2, p. 41.

Changes in Heat Transfer from Turbulent Boundary Layers Interacting with Shock Waves and Expansion Waves

L. H. BACK* AND R. F. CUFFEL†

Jet Propulsion Laboratory, Pasadena, California

Nomenclature

a^* = speed of sound at sonic condition
c = constant
h = heat transfer coefficient
H = enthalpy
M = Mach number
p = pressure
Pr = Prandtl number
q = heat flux to wall
r = channel radius
Re/ft = Reynolds number per ft, $\rho_e u_e/\mu_e$
St = Stanton number
T = temperature
u = velocity parallel to wall
x = distance along wall
z = axial distance
γ = specific heat ratio
μ = viscosity
ρ = density
σ = flow deflection angle
ϕ = energy thickness
ω = viscosity-temperature exponent

Subscripts

aw = adiabatic wall condition
e = freestream condition
0 = reservoir condition
t = stagnation condition
w = wall condition
1 = upstream value
2 = downstream or peak value

Received April 4, 1970; revision received May 21, 1970. This Note presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract NAS 7-100, sponsored by NASA.

* Member Technical Staff, Propulsion Research and Advanced Concepts Section. Associate Fellow AIAA.

† Senior Engineer, Propulsion Research and Advanced Concepts Section. Member AIAA.

Introduction

THIS Note is concerned with heat transfer from turbulent boundary layers in supersonic flows where changes in surface curvature can produce shock waves and expansion waves, e.g., corner flows, or where shock waves generated elsewhere in the flow impinge on the boundary layer. Heat-transfer measurements are presented in these interaction regions and a rather simple method involving the integral form of the energy equation is used to estimate the change in heat transfer that is observed. The prediction is then compared to other experimental data obtained at shock impingement locations.

Measurements

The interaction regions investigated were observed in an axisymmetric supersonic diffuser with air at stagnation pressures of 100, 150, and 200 psia and a stagnation temperature of 1500°R. The measurements included wall static pressures, semilocal wall heat fluxes, coolant-side wall temperatures, and boundary-layer surveys. The heat flux was determined by calorimetric measurements in circumferential coolant passages. The gas-side wall temperature was calculated from the measured wall heat flux and coolant-side wall temperature. A nearly isothermal wall condition was achieved with a wall-to-stagnation temperature ratio of ~0.43. The heat-transfer coefficients, calculated by using the difference between adiabatic wall enthalpy and wall enthalpy, are believed accurate to ~10%. The adiabatic wall enthalpy was calculated using a recovery factor of 0.89, a value which does not seem to differ much in shock impingement regions.[1,2] Reference 3 describes the test apparatus and Ref. 4 the measurement technique as applied to flow through a nozzle.

Boundary-layer surveys were made with a small flattened Pitot tube and an aspirating thermocouple probe with a recovery factor of 0.97. The tips of these probes were 0.005 and 0.010 in. high, respectively, relatively small compared to the boundary-layer thickness. Only the turbulent boundary-layer thicknesses are shown, not the profiles themselves.

Results

Measurements in an expanding flow around a 6° corner and at a shock wave impingement on a turbulent boundary layer are shown in Fig. 1. The expansion fan emanating from the corner is drawn for a uniform, inviscid flow and the nature of the interaction in the impingement region is shown diagrammatically. After impingement, the reflected shock wave is curved away from the wall and this leads to an expanding flow downstream as indicated by the decreasing wall pressures. For the corner flow, a local Prandtl-Meyer expansion at the corner gives a downstream Mach number of 3.59, in close agreement with the measured value. However, a calculation

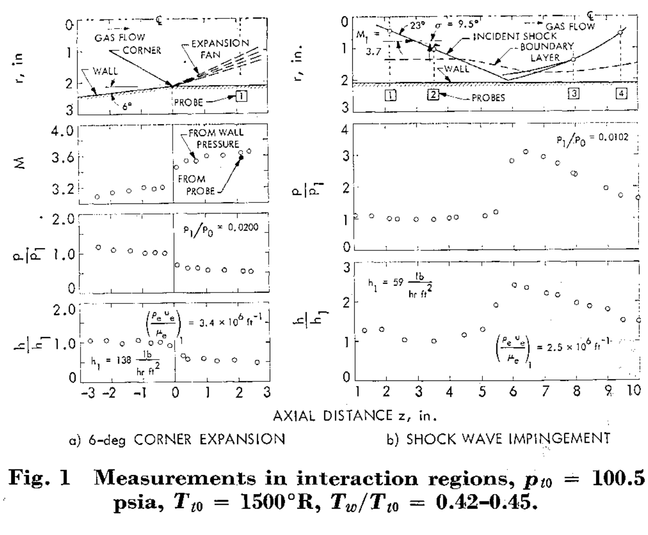

Fig. 1 Measurements in interaction regions, p_{t0} = 100.5 psia, T_{t0} = 1500°R, T_w/T_{t0} = 0.42–0.45.

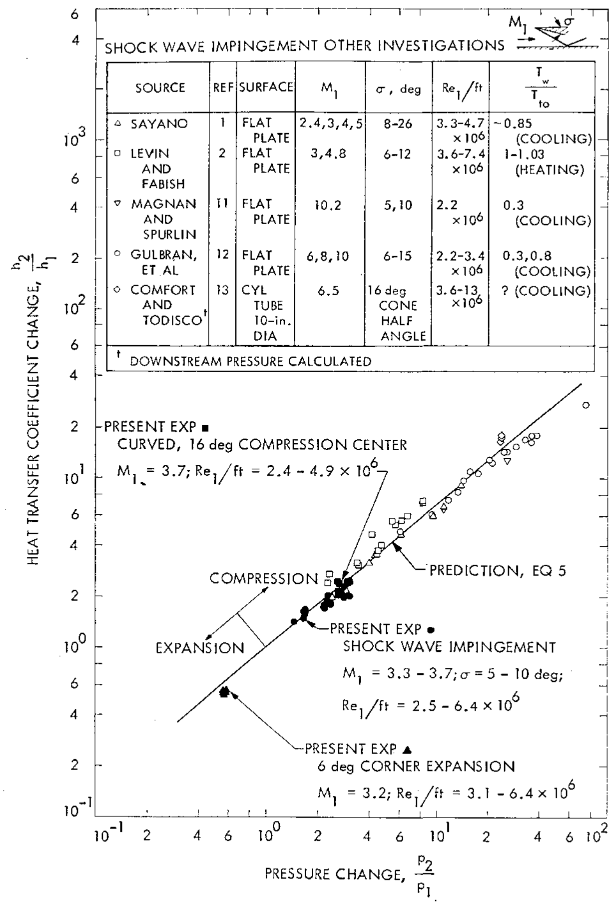

Fig. 2 Turbulent boundary-layer heat-transfer coefficient—pressure relationship at shock wave impingement locations with and without separation and at compression and expansion corners. Present experiments with wall cooling, T_w/T_{t0} = 0.42–0.45.

of the pressure rise for ideal shock wave impingement and reflection from a plane surface, i.e., ignoring the shear layer, overestimates the measured pressure rise of 3.1 by 50%. For either interaction the upstream influence is small, about a boundary-layer thickness in length, and the over-all interaction region is only a few boundary-layer thicknesses long. The velocity and temperature profiles were typical of a turbulent boundary layer. This is not unexpected since, even for the corner expansion, values of the acceleration parameter $K = (\mu_e/\rho_e u_e^2)(du_e/dx)$ are about two orders of magnitude smaller than those associated with laminarization of a turbulent boundary layer in an accelerating flow.[5]

The change in the heat-transfer coefficient across these interactions is deduced by considering the integral form of the energy equation

$$d[\rho_e u_e r^j (H_{t0} - H_w)\phi] = r^j q dx \qquad (1)$$

in conjunction with a specification of the heat transfer by the relation

$$StPr^{2/3} = c(\rho_e u_e \phi/\mu_e)^{-1/4}(T_e/T_{aw})^{(1/4)(3-\omega)} \qquad (2)$$

The energy equation applies to axisymmetric flow with $j = 1$ and to flow over a plane surface with $j = 0$. The basic form of the heat transfer relation, i.e., $St \propto (\rho_e u_e \phi/\mu_e)^{-1/4}$ was suggested earlier by Ambrok,[6] and was in reasonable agreement with experimental measurements in accelerating and decelerating turbulent boundary layers, e.g., Refs. 7–9. The term $(T_e/T_{aw})^{(1/4)(3-\omega)}$ accounts for compressibility effects which reduce the Stanton number below its low speed value (see Ref. 9 for flow through a supersonic nozzle that permits a direct evaluation of compressibility effects because of the relatively large flow speed range, the inlet and exit Mach numbers being 0.06 and 3.7, respectively).

Application of the energy equation to the flows considered indicates that the energy defect in the boundary layer (left side of Eq. 1), may be essentially constant since the interaction regions are relatively short where the boundary layer is turbulent and the heat-transfer surface area consequently is small. For the nearly isothermal wall measurements herein, the change in the heat-transfer coefficient can then be estimated from the heat-transfer relation, recalling the definition of h

$$h = q/(H_{aw} - H_w) = St(\rho_e u_e) \qquad (3)$$

as

$$\frac{h_2}{h_1} = \left[\frac{(\rho_e u_e)_2}{(\rho_e u_e)_1}\right]^{3/4}\left(\frac{\phi_2}{\phi_1}\right)^{-1/4}\left(\frac{T_{e2}}{T_{e1}}\right)^{3/4} = \frac{(\rho_e u_e)_2}{(\rho_e u_e)_1}\left(\frac{T_{e2}}{T_{e1}}\right)^{3/4} \qquad (4)$$

The last equality follows from the first by using Eq. (1). The change in adiabatic wall temperature, generally smaller than the other terms, was neglected. The change in the heat-transfer coefficient is directly proportional to the mass flux change and is dependent to a lesser extent on the static temperature change.

The predicted decrease in the heat-transfer coefficient for the corner flow from Eq. (4), rewritten to facilitate the use of isentropic flow tables, is

$$\frac{h_2}{h_1} = \frac{\left(\frac{\rho_e u_e}{\rho_0 a^*}\right)_2}{\left(\frac{\rho_e u_e}{\rho_0 a^*}\right)_1}\left[\frac{\left(\frac{T_e}{T_0}\right)_2}{\left(\frac{T_e}{T_0}\right)_1}\right]^{3/4} = \frac{0.0851}{0.124}\left(\frac{0.278}{0.328}\right)^{3/4} =$$

$$0.688(0.884) = 0.61$$

This value is a little higher than observed experimentally. For the shock wave impingement, similar agreement is found by using the measured upstream pressure and peak pressure to calculate the quantities in Eq. (4) for an isentropic, external flow, thereby ignoring total pressure losses across the shock waves. The predicted increase in the heat-transfer coefficient is

$$\frac{h_2}{h_1} = \frac{0.165}{0.079}\left(\frac{0.373}{0.270}\right)^{3/4} = 2.09(1.27) = 2.66$$

Additional results obtained at other impingement locations where the interactions were not as strong and in a curved compression corner are shown in Fig. 2. These results encompass more than a two-fold change in Reynolds number

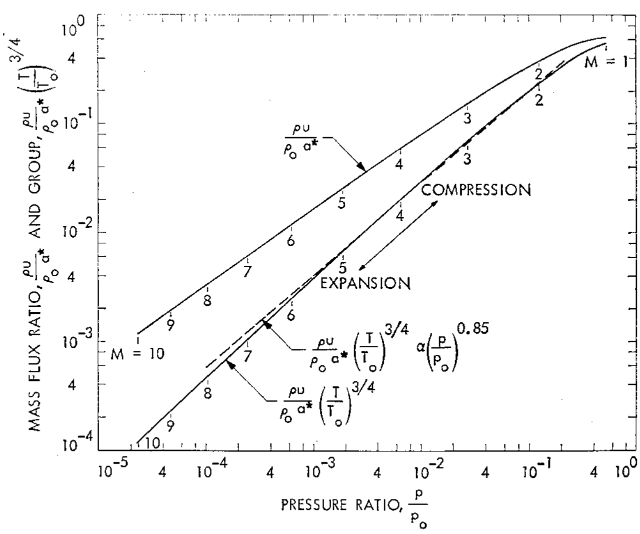

Fig. 3 Isentropic flow variables (γ = 1.4).

going from stagnation pressures of 100 to 200 psia and lower Mach numbers, and also include data with thicker boundary layers upstream of the interactions.

To calculate the distribution of the heat-transfer coefficient, the energy Eq. (1) can be integrated along the wall, since the heat flux is related to the energy thickness by Eqs. (2) and (3), to obtain the energy thickness distribution once the freestream flow variables and wall temperature are specified (see Ref. 3 for such a calculation upstream, within and downstream of some of the interactions presented herein).

Attention is now focused on the interpretation of the prediction when it is applied to other experimental data at shock impingement locations where there appears to be similarity between the measured heat-transfer and pressure changes. For example, Sayano[1] has given the empirical relation $(h_2/h_1) = (p_2/p_1)^{0.8}$ from his results and, more recently Markarian,[10] in reviewing some measurements, suggested a slightly stronger empirical power law dependence with the exponent 0.85. It is useful here to recast Eq. (4) in terms of a pressure change. For isentropic flow, the variation of the mass flux and group from Eq. (4) with pressure ratio is shown in Fig. 3 in nondimensional form. Although over the large Mach number range shown (1–10) there is no particular power law fit, the values agree reasonably well in the supersonic range from about 2–6 with the dashed curve shown. This correspondence implies the following relationship between the change in heat-transfer coefficient and pressure from Eq. (4):

$$h_2/h_1 \cong (p_2/p_1)^{0.85} \tag{5}$$

The prediction from Eq. (5) is compared with experimental measurements in Fig. 2. The agreement is satisfactory, even in the hypersonic flow regime where the prediction might not be expected to agree as well by inference from Fig. 3. This may be partly due to the smaller increase in the group $(\rho_e u_e)_2/(\rho_e u_e)_1[(T_{e_2}/T_{e_1})]^{3/4}$ across shock waves than the corresponding isentropic flow values when the shock waves are stronger, i.e., $\Delta p/p \cong \gamma M\sigma$. For example, for isentropic flow in the hypersonic limit $M_e \rightarrow \infty$, $[(\rho u/\rho_0 a^*)][(T/T_0)]^{3/4} \alpha [(p/p_0)]^n$ where $n = 1/\gamma + (\frac{3}{4})(\gamma - 1/\gamma) = 0.93$ for $\gamma = 1.4$. Also, it could be associated with the correction for compressibility effects as contained in Eq. (2) that partially led to the $T^{3/4}$ dependence in Eq. (4). This correction may be too large in hypersonic flows, e.g., in Eq. (2) as $M_e \rightarrow \infty$, $(T_e/T_{aw}) \rightarrow 0$.

For compression corners, Markarian[10] has shown some results of other investigations in the representation of Fig. 2. Although there is some uncertainty in the structure of the boundary layer just upstream of the corner for some of the data, i.e., transitional or just turbulent in these external flows, and in the resolution of the peak heat transfer downstream, the results do appear to agree with the indication of Eq. (5), as also do the present results obtained with a curved compression corner (Fig. 2).

References

1 Sayano, S., Bausch, H. P., and Donnelly, R. J., "Aerodynamic Heating Due to Shock Impingement on a Flat Plate," SM-41331, Aug. 1962, McDonnell-Douglas Aircraft Co., Santa Monica, Calif.

2 Levin, V. and Fabish, T. J., "Thermal Effects of Shock Wave Turbulent Boundary Layer Interaction at Mach Numbers 3 and 5," 62H-795, Nov. 1962, North American Aviation, Columbus, Ohio.

3 Back, L. H., Cuffel, R. F., and Massier, P. F., "Experimental Convective Heat Transfer and Pressure Distributions and Boundary Layer Thicknesses in Turbulent Flow Through a Variable Cross-Sectional Area Channel," *4th International Heat Transfer Conference Proceedings*, Paris, France, Vol. II FC2.1, 1970, pp. 1–14.

4 Back, L. H., Massier, P. F., and Cuffel, R. F., "Flow Phenomena and Convective Heat Transfer in a Conical Supersonic Nozzle," *Journal of Spacecraft and Rockets*, Vol. 4, No. 8, Aug. 1967, pp. 1040–1047.

5 Back, L. H., Cuffel, R. F., and Massier, P. F., "Laminarization of a Turbulent Boundary Layer in Nozzle Flow—Boundary Layer and Heat Transfer Measurements with Wall Cooling," to be published in ASME *Journal of Heat Transfer*.

6 Ambrok, G. S., "Approximate Solution of Equations for the Thermal Boundary Layer with Variations in Boundary Layer Structure," *Soviet Physics*, Vol. 2 (II), 1957, pp. 1979–1986.

7 Kutateladze, S. S. and Leontev, A. I., "Drag Law in a Turbulent Flow of a Compressible Gas and the Method of Calculating Friction and Heat Exchange," *Akademii Nauk, Belorussk*, SSR, Minsk, Jan. 1961, pp. 23–27; translated and issued by Technical Information and Library Services, Dec. 1961, Ministry of Aviation, London, England.

8 Romanenko, P. N., Leontev, A. I., and Oblivin, A. N., "Investigation on Resistance and Heat Transfer of Turbulent Air Flow in Axisymmetrical Channels with Longitudinal Pressure Gradient," *International Journal of Heat and Mass Transfer*, Vol. 5, June 1962, pp. 541–557.

9 Back, L. H. and Cuffel, R. F., "Turbulent Boundary Layer and Heat Transfer Measurements Along a Convergent-Divergent Nozzle," to be published.

10 Markarian, C. F., "Heat Transfer in Shock Wave-Boundary Layer Interaction Regions," TP 4485, Nov. 1968, Naval Weapons Center, China Lake, Calif.

11 Magnan, J. D., Jr. and Spurlin, C. J., "Investigation of Flow Field Interference Caused by Shock Impingement on a Flat Plate at Mach Numbers of 6, 8, and 10," TR-66-85, April 1966, Arnold Engineering Development Center, Arnold Air Force Base, Tenn.

12 Gulbran, C. E. et al., "Heating in Regions of Interfering Flow Fields, Part III: Two-Dimensional Interaction Caused by Plane Shocks Impinging on Flat Plate Boundary Layers," AFFDL-TR-65-49, Pt. III, March 1967, The Boeing Company, Wright-Patterson Air Force Base, Ohio.

13 Comfort, E. and Todisco, A., "Experimental Investigation of Two Shock Wave Boundary Layer Interaction Configurations," AIAA Paper 69-8, New York, 1969.

Turbulent Boundary-Layer Computations Based on an Extended Mixing Length Approach

Y. Y. Chan*

National Aeronautical Establishment, Ottawa, Canada

IN the recent development of computational methods for turbulent boundary layers[1,2] Prandtl's mixing length concept is again widely used to correlate the turbulent shear stress to the local mean flow of the boundary layer. In the methods of Patankar and Spalding,[3] Beckwith and Bushnell,[4] and Pletcher[5] the mixing length is correlated to the local thickness of the boundary layer and the correlation function for flow passed a flat plate is usually assumed for general uses. The drawback of these methods is in that the flat plate correlation represents a reasonable approximation only for flows with moderate pressure gradients,[6] but is not necessarily true for flows with large pressure gradients, especially for adverse pressure gradients. By directly relating the mixing length to the mean flow, these mean-field methods do not consider the development of the turbulent field explicitly and thus are not very successful in predicting highly nonequilibrium flows. In the method of McDonald and Camarata,[7] these shortcomings are overcome. The mixing length is no longer correlated directly to the mean-flow field of the boundary layer, but is calculated by the integral turbulent kinetic energy equation, thus the history of the turbulent state is considered explicitly and the mixing length correlation is allowed to vary as the turbulent boundary layer develops. This method is similar to the approach of Bradshaw

Received February 16, 1970; revision received April 30, 1970.

* Associate Research Officer, High Speed Aerodynamics Section. Member AIAA.